# Sujets de mémoires – Master theses topics 2024

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## Computational Geometry

**Algorithms on visibility graphs.** A visibility graph of a simple polygon in the plane encodes the visibility relation between the vertices of the polygon. Consider the decomposition of visibility graphs into complete bipartite cliques (see Aronov et al., 1994) and use it to efficiently solve a number of problems of visibility graphs, including maximum matching and all-pairs shortest paths, using the Feder-Motwani framework. Generalizations to pseudo-visibility graphs can also be investigated.

**Closest induced line in $\mathbb{R}^3$.** Describe (and implement) an $O(n \log n)$ time algorithm for finding the closest line to a given query point, among all lines through pairs of points of a set in $\mathbb{R}^3$. See Cardinal-Ooms, JoCG 2022.

## Data Structures

**Function inversion.** Describe (and implement) the Fiat-Naor function inversion scheme (SICOMP 1999) in cryptography, and give one detailed application to a geometric data structure. See our recent SOSA paper for examples of applications.

## Polyhedral combinatorics and linear programming

**Rainbow and facet-Hamiltonian cycles in flip graphs.** Triangulations of convex polygons have a natural and elegant structure involving convex polytopes called associahedra. Hamiltonian and rainbow cycles are different families of cycles on the graphs of associahedra. Describe and implement the rainbow cycles from Kleist et al. and take the opportunity to present the various (Coxeter) types of permutahedra and associahedra and their Hamiltonian cycles.

**Shortest paths on polymatroids** Base polytopes of polymatroids, also known as generalized permutohedra, are polytopes whose edges are parallel to a

vector of the form $e_i - e_j$. We consider the following computational problem: Given two vertices of a generalized permutohedron P, find a shortest path between them on the skeleton of P. This captures many known flip distance problems. Even though this problem is NP-hard, there is an exact polynomial-time algorithm for the special case of flip distance between two acyclic orientations of any linear hypergraph. Analyze and implement this algorithm. Based on a recent paper with Raphael Steiner.

## Graph algorithms

**Discrete bulk reconstruction.** Consider the problem posed recently by Aaronson and Pollack in JHEP, motivated by the AdS/CFT correspondence, to realize entropies of circular intervals as minimum cuts in a planar graph. Implement their solution, using a universal graph of quadratic size, and consider the optimization problem in which we seek to minimize the complexity of the graph (number of vertices, edges, etc.)